ORTHOTROPIC AND COMPOSITE MATERIAL MODELING. ANALYSIS OF STRESS AND STRAIN STATE IN THE ORTHOTROPIC LAYER AND IN THE MULTILAYER LAMINATE.

1 Introduction

1.1 Orthotropic materials

Occurrence of three mutually perpendicular planes of symmetry of the material properties is common in engineering practice. Thus the number of independent coefficients of the constitutive matrix, describing relation between stress and strain, reduces in this case to 9 and the material is called *orthotropic*.

Material constants are set in a way that corresponds to the Young's modulus E, Poisson's ratio ν and shear modulus G for isotropic case. Stress - strain relation takes the following form:

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{E_{11}} & -\dfrac{\nu_{21}}{E_{22}} & -\dfrac{\nu_{31}}{E_{33}} & 0 & 0 & 0 \\ -\dfrac{\nu_{12}}{E_{11}} & \dfrac{1}{E_{22}} & -\dfrac{\nu_{32}}{E_{33}} & 0 & 0 & 0 \\ -\dfrac{\nu_{13}}{E_{11}} & -\dfrac{\nu_{23}}{E_{22}} & \dfrac{1}{E_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix}. \tag{1.1}$$

Constants E_{11}, E_{22} and E_{33} are called tensile strenght moduli (Young's moduli) in directions 1, 2 and 3 respectively. G_{12}, G_{23} and G_{31} are shear moduli in corresponding planes, and ν_{ij} denotes Poisson's ratios. Constants appearing in the constitutive matrix must fulfill additional relations resulting from its symmetry:

$$\frac{\nu_{12}}{E_{11}} = \frac{\nu_{21}}{E_{22}}, \quad \frac{\nu_{13}}{E_{11}} = \frac{\nu_{31}}{E_{33}}, \quad \frac{\nu_{23}}{E_{22}} = \frac{\nu_{32}}{E_{33}}. \tag{1.2}$$

In result, the consitutive law for the tridimentional ortothropic material consists of 9 independent material constants: E_{11}, E_{22}, E_{33}, ν_{21}, ν_{31}, ν_{32}, G_{12}, G_{23}, G_{31}. Moduli E_{ii} are obtained from proper tensile tests performed along the main axes of orthotrophy. Obtaining shear moduli G_{ij} demands a proper shear tests, whereas Poisson's ratios ν_{ij} are determined by the following ratio: $-\frac{\varepsilon_{jj}}{\varepsilon_{ii}}$ calculated for a sample with only tensile stress σ_{ii} acting on it.

1.2 Strains and stresses in the orthotropic layer

Many composite structures consist of layers with the orthotropic properties. In the analysis of the stress - strain relation in the orthotropic layer we can assume the plane stress state. Assuming that thin, orthotropic layer with the principal orthotropic directions 1 and 2 is located in XY plane with the force acting only in the same plane (fig. 1.1) we obtain $\tau_{xz} = \tau_{yz} = \sigma_{zz} = 0$ as well as $\tau_{31} = \tau_{23} = \sigma_{33} = 0$. From the equation 1.1 we obtain:

$$\gamma_{31} = 0, \quad \gamma_{23} = 0, \quad \varepsilon_{33} = -\frac{\nu_{13}}{E_{11}}\sigma_{11} - \frac{\nu_{23}}{E_{22}}\sigma_{22}. \tag{1.3}$$

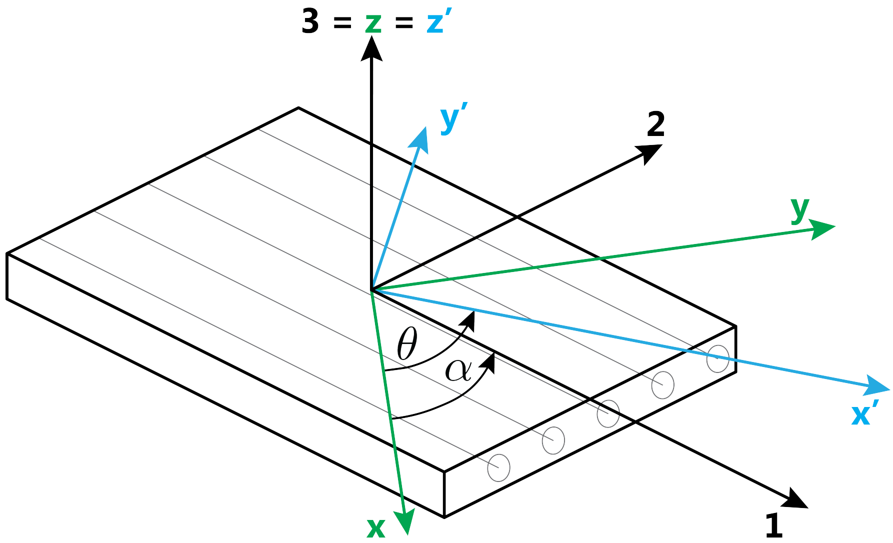

Figure 1.1: Coordinate systems in the orthotropic layer.

Relations between stresses and strains reduce to the following form:

$$\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{bmatrix} = \begin{bmatrix}\frac{1}{E_{11}} & -\frac{\nu_{21}}{E_{22}} & 0\\ -\frac{\nu_{12}}{E_{11}} & \frac{1}{E_{22}} & 0\\ 0 & 0 & \frac{1}{G_{12}}\end{bmatrix}\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{bmatrix}, \qquad \begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}\frac{E_{11}}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} & 0\\ \frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}} & \frac{E_{22}}{1-\nu_{12}\nu_{21}} & 0\\ 0 & 0 & G_{12}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{bmatrix}. \tag{1.4}$$

Matrices from equations 1.4 are symmetrical, therefore only 4 independtent material constants are neccesary to its full determination, for example: E_{11}, E_{22}, ν_{12} and G_{12}.

In the engineering literature the constants are usually assigned in a specyfic way: $E_{11} > E_{22}$ thus coefficient ν_{12} is called *major* and ν_{21} *minor* Poisson's ratio.

Relations 1.4 correspond to the case in which directions 1 and 2 are the principal orthotropic directions of the layer. However, in many cases axes of the selected coordinate system are not colinear with the material's principal orthotropic directions. Relations between stress and strain states in two arbitrary cartesian coordinate systems, rotated relative to each other by the angle θ, are as follows:

$$\begin{bmatrix}\sigma_{x'x'}\\ \sigma_{y'y'}\\ \tau_{x'y'}\end{bmatrix} = [T]\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}, \qquad \begin{bmatrix}\varepsilon_{x'x'}\\ \varepsilon_{y'y'}\\ \frac{1}{2}\gamma_{x'y'}\end{bmatrix} = [T]\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \frac{1}{2}\gamma_{xy}\end{bmatrix}. \tag{1.5}$$

Transformation matrix $[T]$ is given by:

$$[T] = \begin{bmatrix}\cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta\\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta\\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta-\sin^2\theta\end{bmatrix}. \tag{1.6}$$

It is worth noting that relations 1.5 correlate stresses (strains) in two arbitrary, rotated by angle θ, coordinate systems and apply both to isotropic and anisotropic medium. The angle θ is measured counterclockwise from x, y axes to x', y' axes.

To show relations between components of stress state and components of strain state for the orthotropic layer we assume that the principal orthotropic directions 1 and 2 are pivoted by α from x and y directions of the cartesian coordinate system (fig. 1.1). General stress - strain relation for the orthotropic layer in an arbitrary coordinate system, determined by the α, has the following form:

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix} = \begin{bmatrix}S^*_{11} & S^*_{12} & S^*_{16}\\ S^*_{12} & S^*_{22} & S^*_{26}\\ S^*_{16} & S^*_{26} & S^*_{66}\end{bmatrix}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}, \qquad \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}Q^*_{11} & Q^*_{12} & Q^*_{16}\\ Q^*_{12} & Q^*_{22} & Q^*_{26}\\ Q^*_{16} & Q^*_{26} & Q^*_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix}. \tag{1.7}$$

S^*_{ij} are given by following equations:

$$\begin{aligned}
S^*_{11} &= S_{11}c^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}s^4,\\
S^*_{12} &= S_{12}c^4 + (S_{11} + S_{22} - S_{66})s^2c^2 + S_{12}s^4,\\
S^*_{22} &= S_{22}c^4 + (2S_{12} + S_{66})s^2c^2 + S_{11}s^4,\\
S^*_{66} &= S_{66}c^4 + 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})s^2c^2 + S_{66}s^4,\\
S^*_{16} &= (2S_{11} - 2S_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})s^3c,\\
S^*_{26} &= (2S_{12} + S_{66} - 2S_{22})sc^3 - (2S_{12} + S_{66} - 2S_{11})s^3c,
\end{aligned} \tag{1.8}$$

where $s = \sin\alpha$, $c = \cos\alpha$ and S_{ij} coefficients are equal to:

$$S_{11} = \frac{1}{E_{11}}, \qquad S_{12} = \frac{-\nu_{21}}{E_{22}}, \qquad S_{22} = \frac{1}{E_{22}}, \qquad S_{66} = \frac{1}{G_{12}}. \tag{1.9}$$

Q^*_{ij} are given by following equations:

$$\begin{aligned}
Q^*_{11} &= Q_{11}c^4 + (Q_{12} + 2Q_{66})s^2c^2 + Q_{22}s^4,\\
Q^*_{12} &= Q_{12}c^4 + (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}s^4,\\
Q^*_{22} &= Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 + Q_{11}s^4,\\
Q^*_{66} &= Q_{66}c^4 + (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}s^4,\\
Q^*_{16} &= (Q_{11} - Q_{12} - 2Q_{66})sc^3 - (Q_{12} - Q_{22} - 2Q_{66})s^3c,\\
Q^*_{26} &= (Q_{12} + Q_{66} + 2Q_{22})sc^3 - (Q_{11} + Q_{12} - 2Q_{66})s^3c,
\end{aligned} \tag{1.10}$$

where Q_{ij} are equal to:

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}, \quad Q_{66} = G_{12}. \tag{1.11}$$

It is noteworthy that for $\alpha \neq k\frac{\pi}{2}$ all of the coefficients of the matrices $[Q^*]$ and $[S^*]$ are nonzero and full coupling between the stress state components and the strain state components exists. Simple tension leads only to elongation in the direction of acting force and to compression in the transverse direction. Shear deformation does not occur. Nonetheless, in the case of tension in the different, arbitrary chosen direction, we will also observe change in the angles. On that account, in general, when the axes x, y are not collinear with the principal orthotropy directions, then the principal stress directions are not collinear with the principal strain directions.

1.3 Basics of laminate mechanics

While analysing the properties of a laminate consisting of many closely connected orthotropic layers, we often assume that it behaves similarly to a single layer. The simpliest mathematical relation can be obtained for so-called *thin laminates*. The thickness of individual layers as well as the overall thickness are much smaller compared to the other dimensions. In this case, the stress components perpendicular to the layers might be neglected. It can also be assumed that the strain countinuity is maintained during the transision between the layers. Due to differences in the properties of individual layers, the stress field is discontinuous between layers.

If we assume small deflections only and Kirchoff's hypothesis, stating that straight lines normal to the laminate surface remain normal to it after deformation is preserved, then the deformation of laminate subjected to force and moment is described by:

$$
\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_x \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B} & \boldsymbol{D} \end{bmatrix}^{-1} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}, \tag{1.12}
$$

where : $\varepsilon_x^0,\ \varepsilon_y^0,\ \gamma_{xy}^0$ - components of the strain state in the laminate's mid-layer,

$\kappa_x = \dfrac{\partial^2 w_0}{\partial x^2},\ \kappa_y = \dfrac{\partial^2 w_0}{\partial y^2},\ \kappa_{xy} = \dfrac{\partial^2 w_0}{\partial x \partial y}$ - curvature of the laminate's surface,

w_0 - deflection of the laminate's mid-surface,

$N_x,\ N_y,\ N_{xy},\ M_x,\ M_y,\ M_{xy}$ - internal forces in the laminate (fig. 1.2),

$\boldsymbol{A}$ - extensional stiffness matrix of the laminate,

$\boldsymbol{B}$ - coupling stiffness matrix of the laminate,

$\boldsymbol{D}$ - bending stiffness matrix of the laminate.

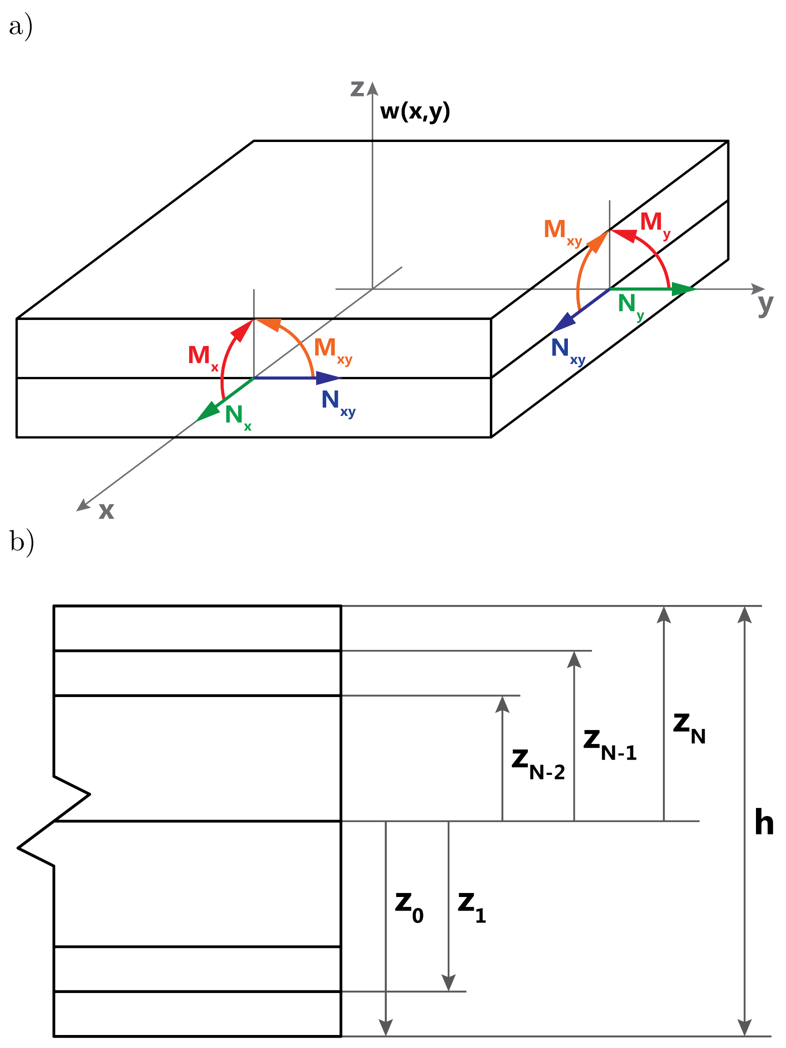

Figure 1.2: Laminate consisting of multiple orthotropic layers: a) internal forces, b) laminate lay-up.

The matrices **A**, **B**, **D** characterize laminate behaviour. They are symmetrical and have dimensions 3 x 3. They depend on the orientation of the layer and its location in the laminate:

$$\boldsymbol{A}_{ij} = \sum_{k=1}^{N} Q_{ij}^{*k}(z_k - z_{k-1}), \quad \boldsymbol{B}_{ij} = \frac{1}{2}\sum_{k=1}^{N} Q_{ij}^{*k}(z_k^2 - z_{k-1}^2), \quad \boldsymbol{D}_{ij} = \frac{1}{3}\sum_{k=1}^{N} Q_{ij}^{*k}(z_k^3 - z_{k-1}^3), \tag{1.13}$$

where : z_k - distance of k-th layer from the mid-surface,

N - number of layers in the laminate,

Q_{ij}^{*k} - stiffness matrix of k-th layer transformed to the xy system.

Above relations are simplified in the case of symmetrical or anti-symmetrical layering of the laminate with respect to the mid-surface. Symmetry or anti-symmetry of the laminate applies not only to the angle of laminating, but also to the mechanical properties and thickness of the layer.

Based on equations 1.3 in case of the symmetry, it can be concluded that all coefficients of the coupling stiffness matrix $\boldsymbol{B}$ are equal to zero, and thus lead to significant simplification of the mathematical description of the laminate.

2 Examplorary problems

2.1 Unidirectional tension of the orthotropic layer

An orthotropic layer has the following properties: $E_{11} = 2 \cdot 10^5\ MPa$, $E_{22} = 0.5 \cdot 10^5\ MPa$, $\nu_{12} = 0.25$, $G_{12} = 2 \cdot 10^4\ MPa$. Find strain components in xy system resulting from tension in x direction - $\sigma_x = 10\ MPa$ as a function of the magnitude of α (fig. 2.1).

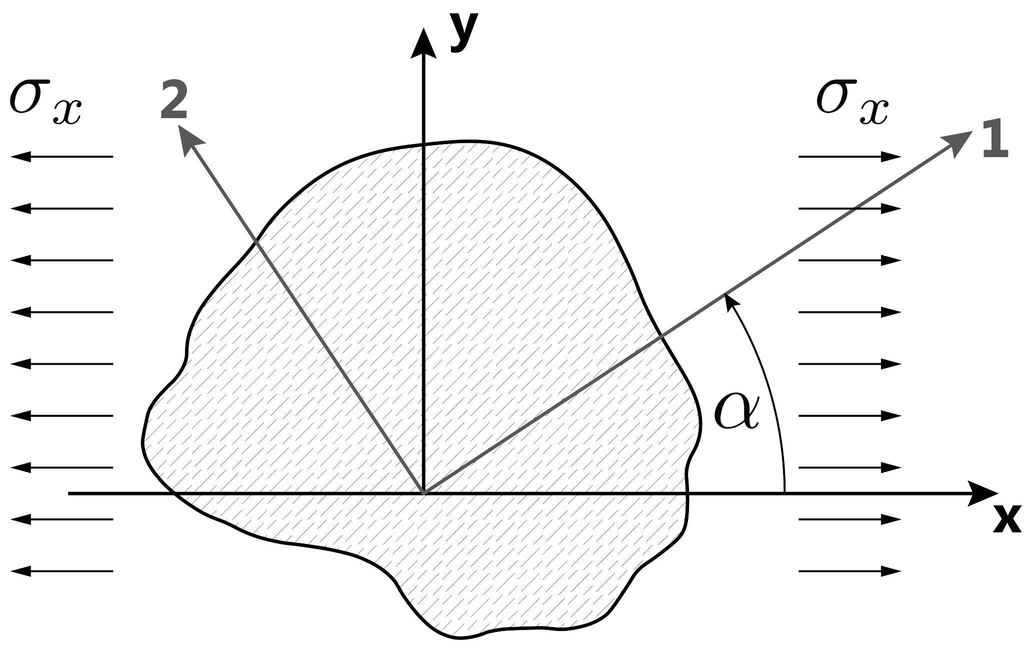

Figure 2.1: Orthotropic layer subjected to tension in an arbitrary direction α.

Solution:
Utilizing relations 1.7 - 1.11 we can obtain:

$$\begin{aligned} \varepsilon_{xx} &= S_{11}^{*}\sigma_{xx}, \\ \varepsilon_{yy} &= S_{12}^{*}\sigma_{xx}, \\ \gamma_{xy} &= S_{66}^{*}\sigma_{xx}, \end{aligned} \tag{2.1}$$

where:

$$
\begin{aligned}
S^*_{11} &= S_{11}c^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}s^4,\\
S^*_{12} &= S_{12}c^4 + (S_{11} + S_{22} - S_{66})s^2c^2 + S_{12}s^4,\\
S^*_{16} &= (2S_{11} - 2S_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})s^3c,
\end{aligned}
\qquad (2.2)
$$

$$S_{11} = \frac{1}{E_{11}}, \qquad S_{22} = \frac{1}{E_{22}}, \qquad S_{12} = \frac{-\nu_{21}}{E_{22}} = \frac{-\nu_12}{E_{11}}, \qquad S_{66} = \frac{1}{G_{12}}.$$

By substituting given material's properties values and constant tensile stress σ_{xx} we obtain results for different magnitudes of α (table 2.1).

$\alpha[°]$	0	5	15	30	45	60	75	85	90
$\varepsilon_{xx}[‰]$	0.500	0.528	0.741	1.297	1.812	2.047	2.040	2.006	2.000
$\varepsilon_{yy}[‰]$	-0.125	-0.142	-0.266	-0.547	-0.688	-0.547	-0.266	-0.142	-0.125
$\gamma_{xy}[‰]$	0.000	-0.323	-0.862	-1.137	-0.750	-0.162	-0.112	-0.062	0.000

Table 2.1: Strains in the orthotropic layer as a function of α.

Same results can be obtained by FEM analysis.

2.2 Unidirectional tension of the multilayer laminate

Determine the response of the 6-layer composite plate to the unidirectional tensile force $N_x = 10\frac{N}{mm}$. The thickness of each layer is equal to 0.125 mm, and the laminating angles are as follows: [45°, −45°, 45°, −45°, 45°, −45°].

Dimmensions of the plate:

- length $L_x = 100\ mm$
- width $L_y = 100\ mm$

Each layer is made from epoxy resin with graphite fibers with the following properties:

- Young's moduli $E_{11} = 211000\ MPa, \quad E_{21} = 5300\ MPa$,
- shear modulus $G_{12} = 2600\ MPa$,
- Poisson's ratio $\nu_{12} = 0.25$

<u>Solution:</u> Equation 1.12 in this case takes the following form:

$$
\begin{bmatrix} \varepsilon^0_x \\ \varepsilon^0_y \\ \gamma^0_{xy} \\ \kappa_x \\ \kappa_x \\ \kappa_{xy} \end{bmatrix}
=
\begin{bmatrix}
1.348\cdot 10^{-4} & -1.216\cdot 10^{-4} & 0 & 0 & 0 & -1.690\cdot 10^{-5}\\
-1.216\cdot 10^{-4} & 1.348\cdot 10^{-4} & 0 & 0 & 0 & -1.690\cdot 10^{-5}\\
0 & 0 & 2.685\cdot 10^{-5} & -1.690\cdot 10^{-5} & -1.690\cdot 10^{-5} & 0\\
0 & 0 & -1.690\cdot 10^{-5} & -2.876\cdot 10^{-3} & -2.594\cdot 10^{-5} & 0\\
0 & 0 & -1.690\cdot 10^{-5} & -2.594\cdot 10^{-3} & 2.876\cdot 10^{-3} & 0\\
-1.690\cdot 10^{-5} & -1.690\cdot 10^{-5} & 0 & 0 & 0 & 5.728\cdot 10^{-4}
\end{bmatrix}
\begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \qquad (2.3)
$$

Thus the strain state of the model can be calculated:

$$
\begin{bmatrix} \varepsilon^0_x \\ \varepsilon^0_y \\ \gamma^0_{xy} \\ \kappa_x \\ \kappa_x \\ \kappa_{xy} \end{bmatrix}
=
\begin{bmatrix} 1.348\cdot 10^{-3} \\ -1.216\cdot 10^{-3} \\ 0 \\ 0 \\ 0 \\ -1.690\cdot 10^{-4} \end{bmatrix}. \qquad (2.4)
$$

Given the strain state is constant, the maximal displacements of the model can be calculated analytically:

$$\begin{aligned} U_x &= \varepsilon_x^0 L_x = 0.135\ mm, \\ U_y &= \varepsilon_y^0 L_y = -0.122\ mm, \\ U_z &= \kappa_{xy} \frac{L_x L_y}{2} = -0.845\ mm. \end{aligned} \tag{2.5}$$

3 Typical numercial routine

3.1 Unidirectional tension of the orthotropic layer

<u>Preprocesor:</u>

A. Creation of the shape of the analyzed field (e.g. square with a side length 100 mm),

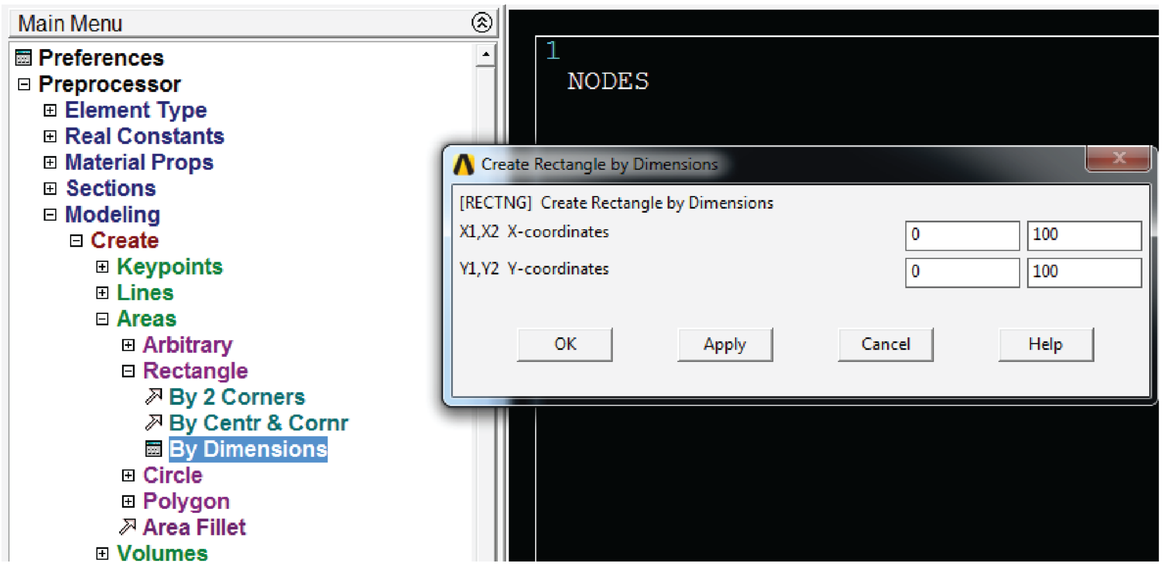

Figure 3.1: Creation of the square.

B. Material properties definition ($E_{11}, E_{22}, \nu_{12}, G_{12}$).

According to equations 1.4 constant ν_{12} should be considered as major Poisson's ratio. We provide that condition during the material properties definition.

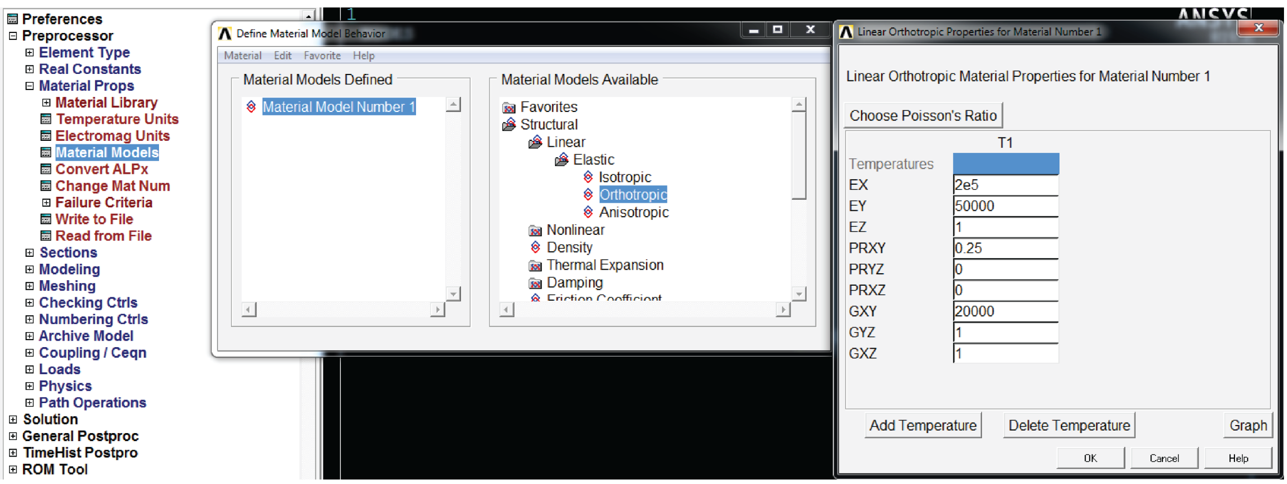

Figure 3.2: Defining properties of the orthotropic material.

C. Type of the finite element selection (*Structural Solid*, e.g. *Plane183*),

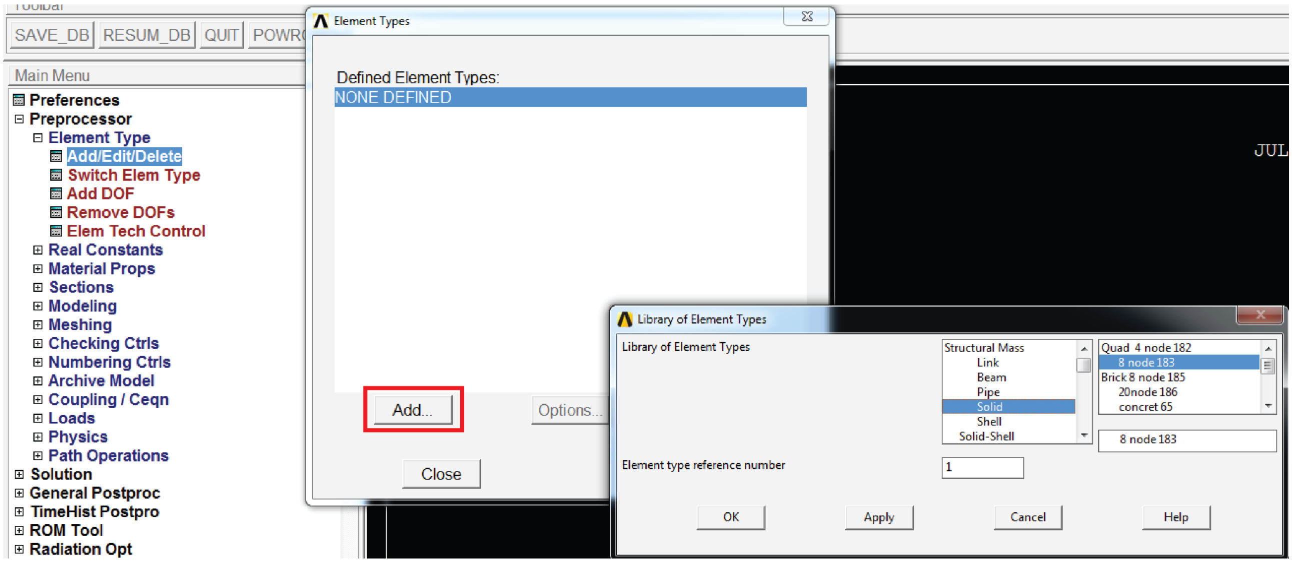

Figure 3.3: Selecting the *Plane183* finite element.

D. Finite element mesh generation.

As we are analyzing constant stress and strain states in the whole area of the model, very fine mesh is not neccesary. Length of the all edges can be changed via:

Mesh Tool $\longrightarrow$ *Size Control* $\longrightarrow$ *Global* $\longrightarrow$ *Set* (**fig. 3.4**).

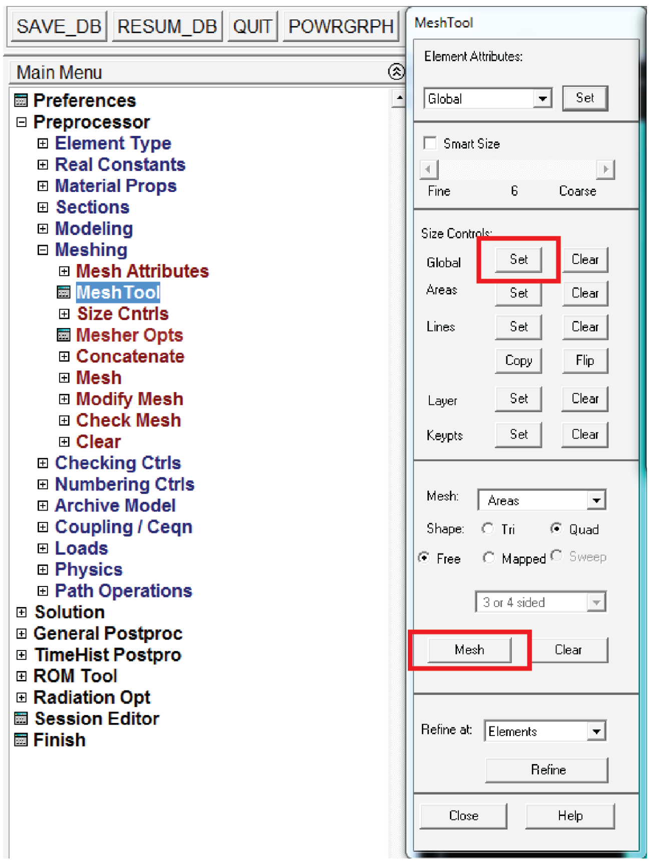

Figure 3.4: Meshing the model. Size of the element was set with the Global Size Control command.

E. Definition of principal ortohtrophy directions.

The orthotropy directions coincide with the directions of so-called *Element Coordinate System*. If the orientation of this system is not specifically determined, it remains coincident with global cartesian system (No. 0).

Directions of element coordinate systems can be checked graphically:

PlotCtrl $\longrightarrow$ *Symbols* **(fig. 3.5)**.

Figure 3.9**A** shows initial directions of element coordinate systems. As can be seen (triad in the center of the element) they are consistent with the xy coordinate system.

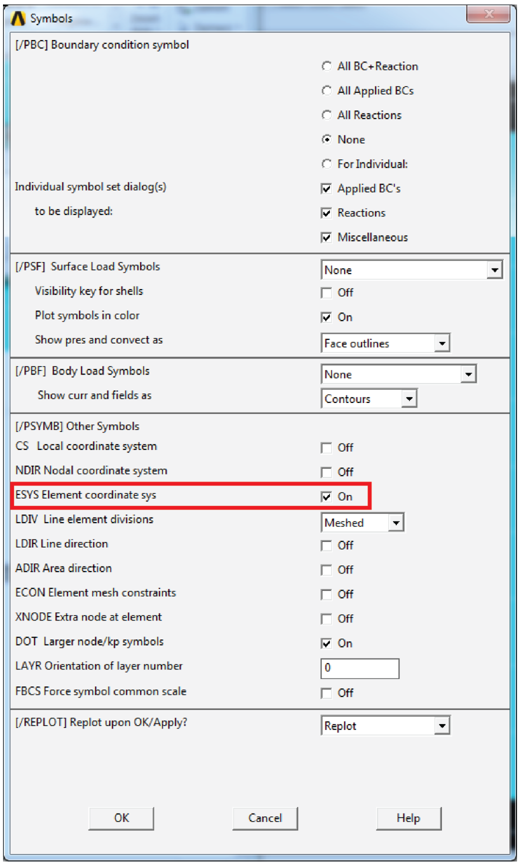

Figure 3.5: Enabling presentation of the element coordinate system.

Element coordinate systems can be modified for individual elements. For this purpose we must define local coordinate rotated to be consistient with the prinicpal orthotropy directions. To do that we are going to utilize *Working Plane* (WP). It needs to be rotated first:

WorkPlane $\longrightarrow$ *Offset WP by Increments* **(fig. 3.6)**.

After rotation we can create local coordinate system:

WorkPlane $\longrightarrow$ *Local Coordinate Systems* $\longrightarrow$ *Create Local CS* $\longrightarrow$ *At WP Origin* **(fig. 3.7)**.

To modify coordinate system assigned to the elements we use the *EMODIF* command:

Modelling $\longrightarrow$ *Move/Modify* $\longrightarrow$ *Elements* $\longrightarrow$ *ModifyAttrib* **(fig. 3.8)**.

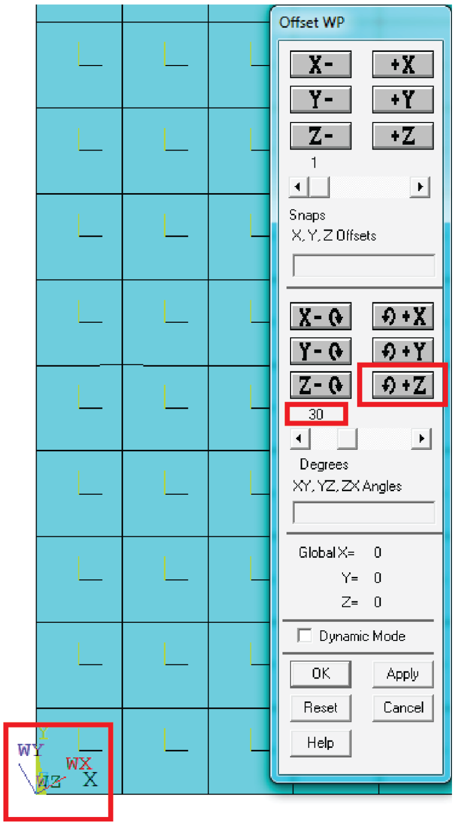

Figure 3.6: Rotating WP by 30° (angle of orthotropy direction in this example). Notice the proper direction of rotation.

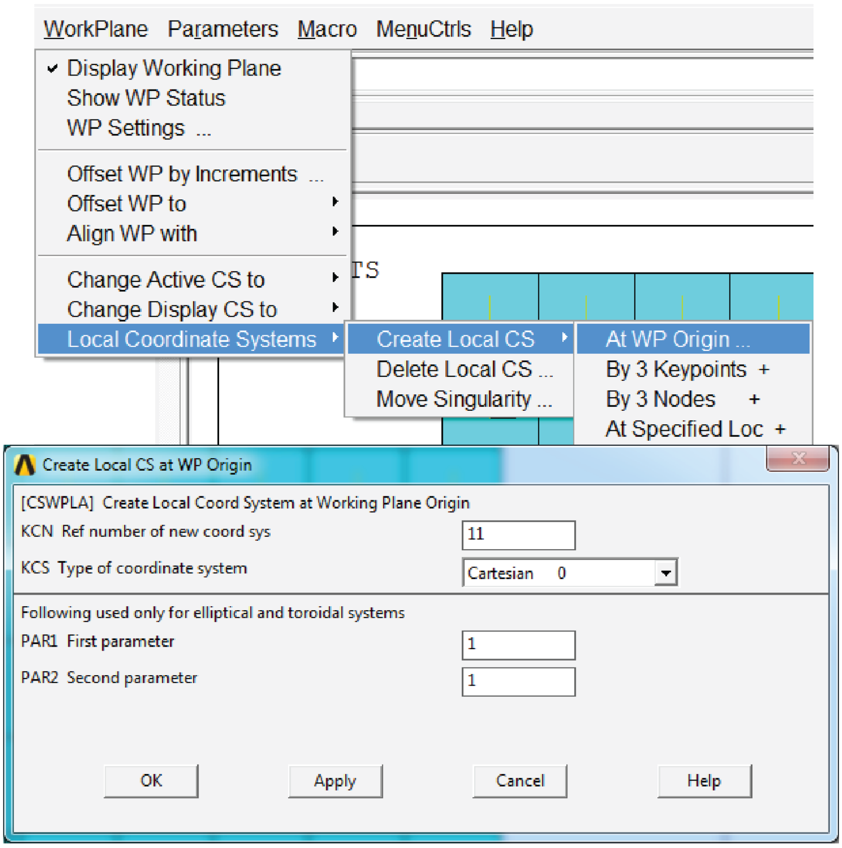

Figure 3.7: Creation of the local coordinate system.

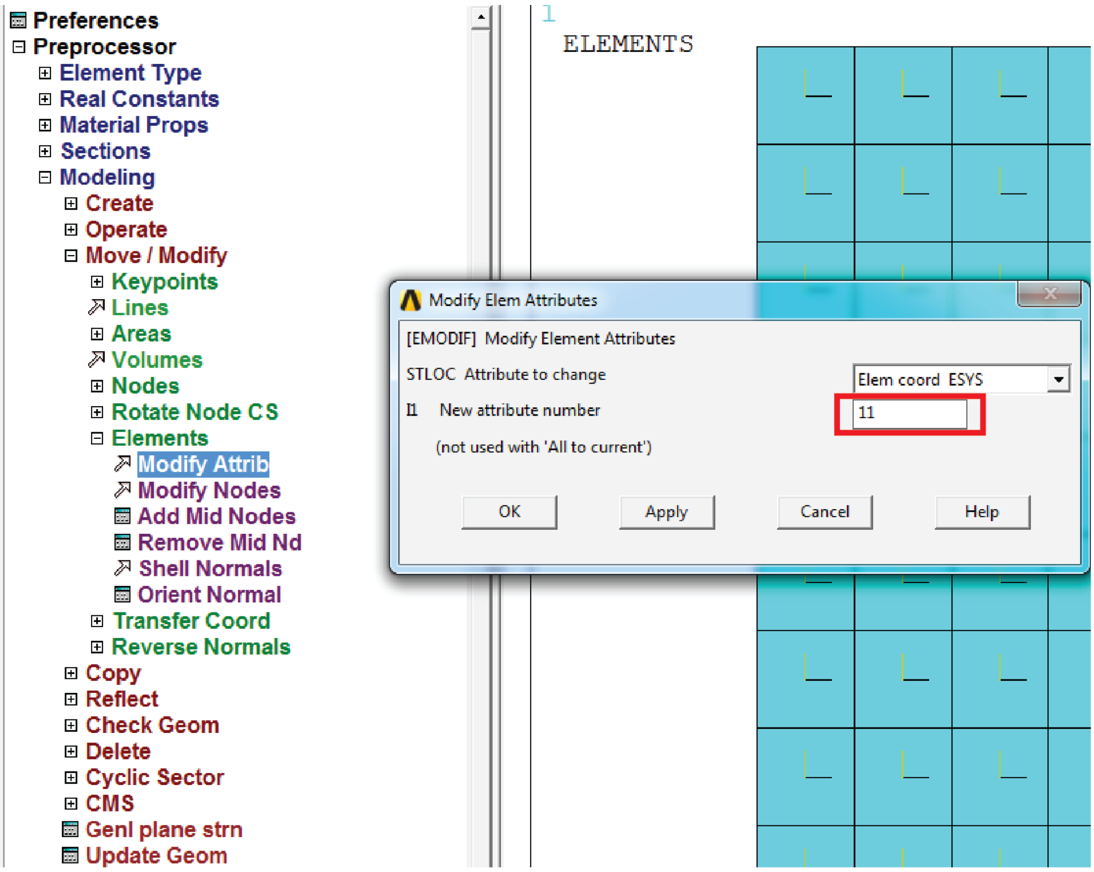

Figure 3.8: Modification of the element coordinate system for all elements.

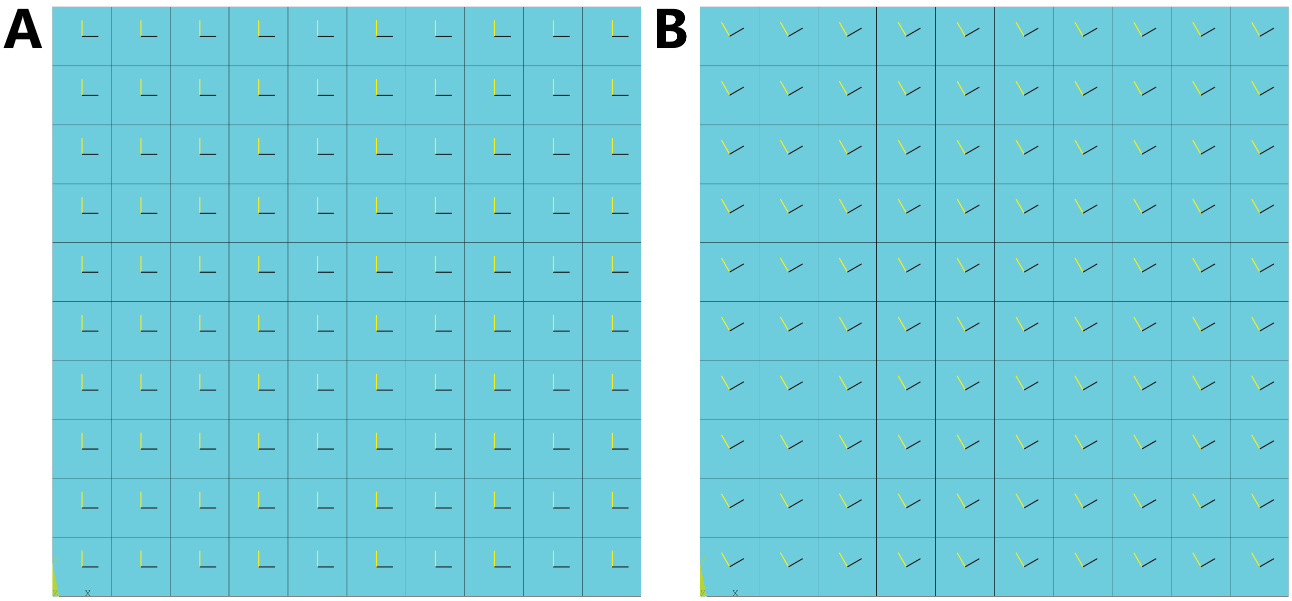

Figure 3.9: Element coordinate systems before (**A**), and after (**B**) the direction modification.

F. Application of the constraints (statically determinate) and the loads acting on the model (pressure $10\frac{N}{mm}$ on two of the model edges).

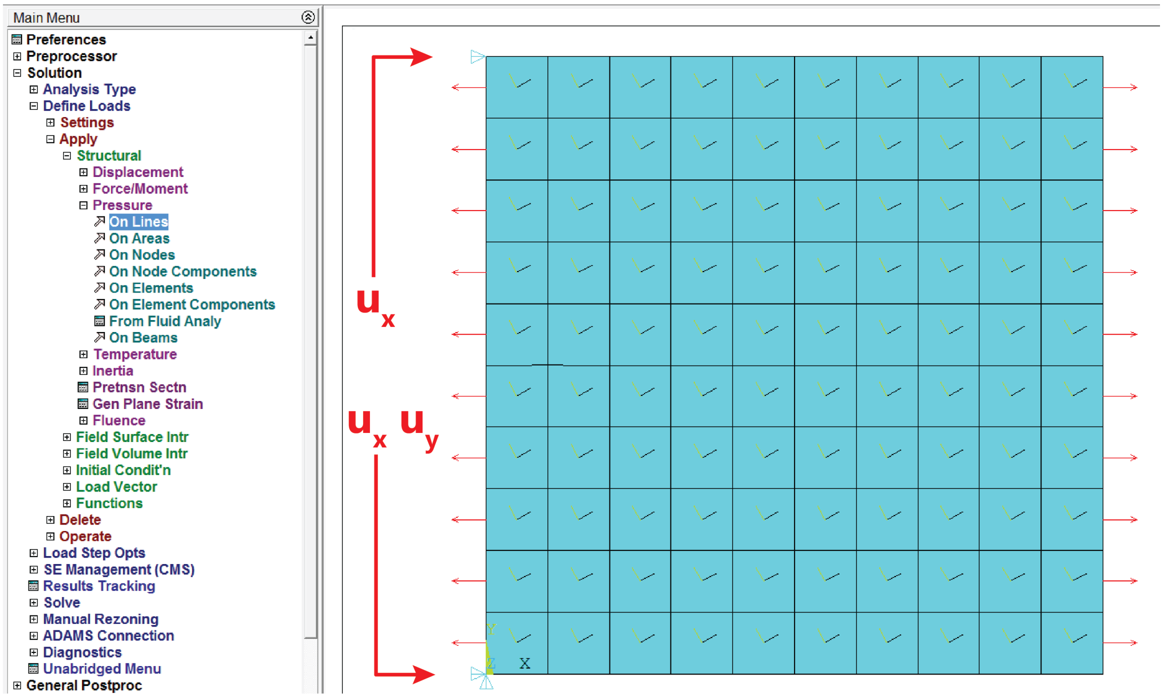

Figure 3.10: Applied boundary condition. Two keypoints were constrained in a statically determinate manner and the pressure was applied on two vertical edges of the model.

Solution: Running the solution: *Solve* $\longrightarrow$ *Current Load Step*.

General Postprocessor:

Evaluation of the results and preparation of the plots. It can be confirmed that the components of stress and strain states are constant in the whole analysis field. In addition results agree with those from the table 2.1 obtained for the 30°.

Although the specimen is subjected to tension only in the horizontal direction, we can observe shear deformation.

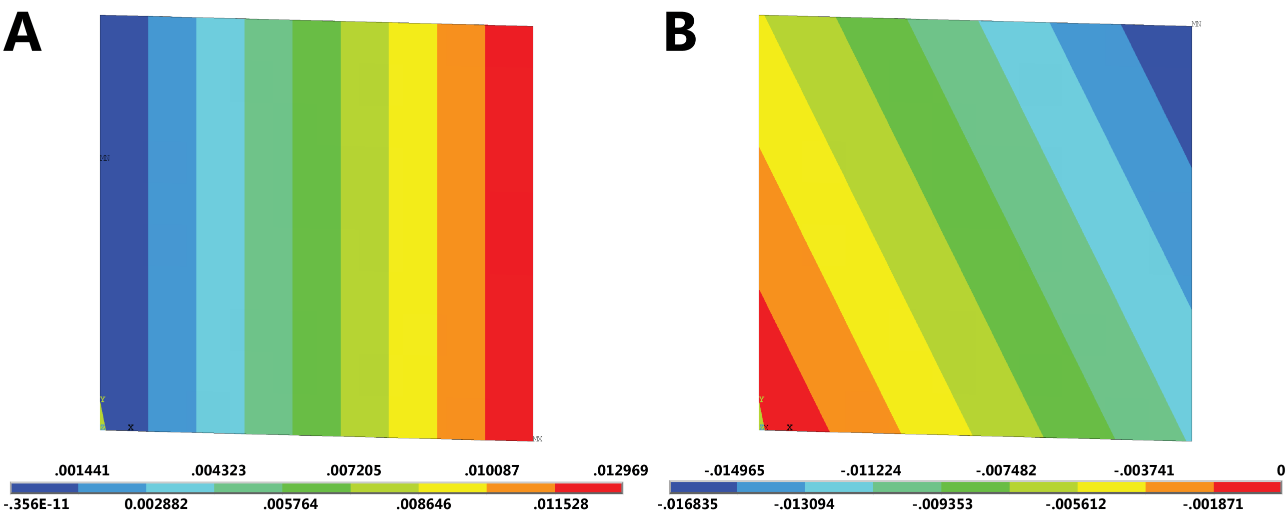

Figure 3.11: Results obtained from analysis. **A** - displacements in the x direction. **B** - displacements in the y direction.

It is unjustified to use Huber-Mises reduced stress theory for the orthotropic material as a measure of effort of the material. ANSYS software allows to use, in relation to anisotropic materials, different yield criterions: maximum stress criterion, maximum strain criterion or Tsai-Wu criterion. It is necessary to provide the relevant material strength data:

Preprocessor ⟶ *Material Props* ⟶ *FailureCrit* ⟶ *Add/Edit* **(fig. 3.12)**.

Most important of those data are: ultimate tensile and compressive strength in principal orthotropy directions and ultimate shear strengths. Plots show distribution of strenght index, where 0 corresponds to the stress-free state and 1 - the ultimate strenght of the orthotropic material in particular stress state.

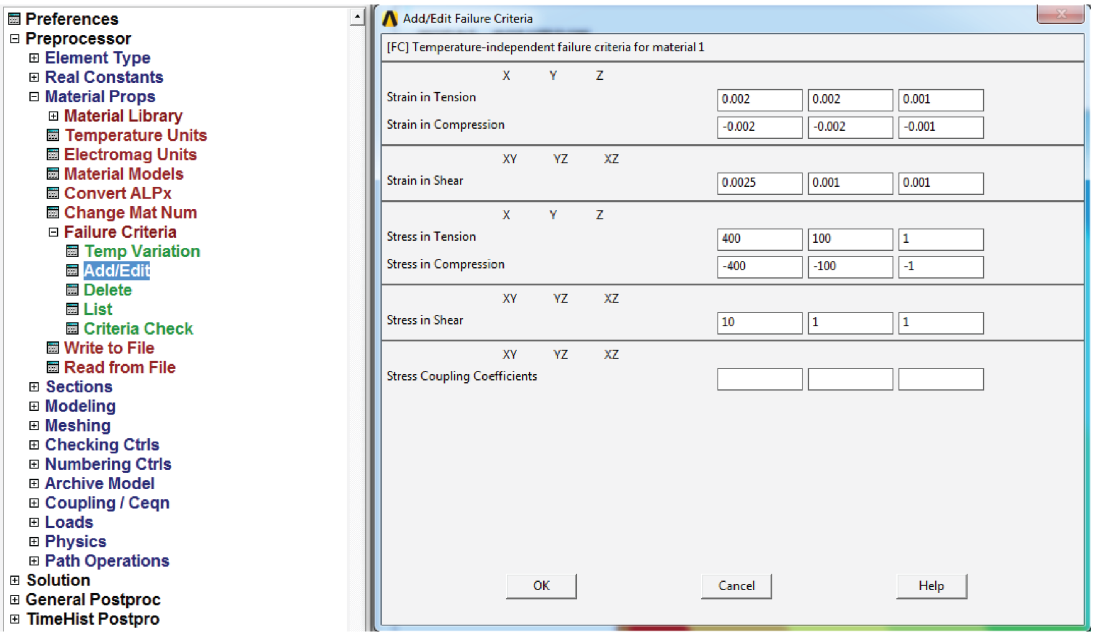

Figure 3.12: Addition of the critical material values.

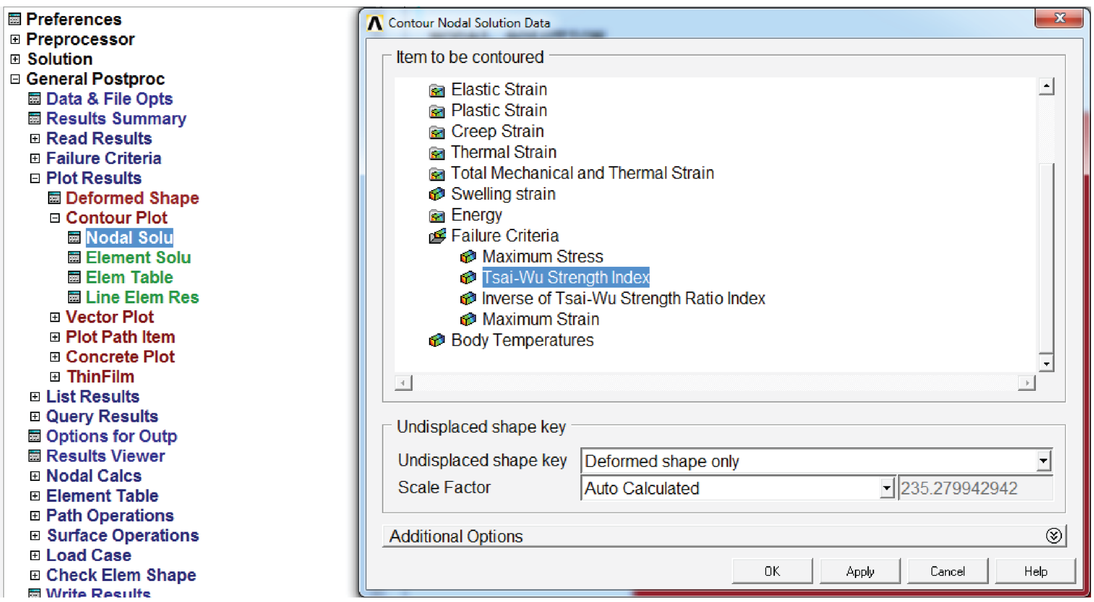

Figure 3.13: Avaiable yield criterions for the orthotropic materials.

3.2 Unidirectional tension of the multilayer laminate

Preprocesor:

A. Creation of geometrical model of the plate (square with a side length 100 *mm*),

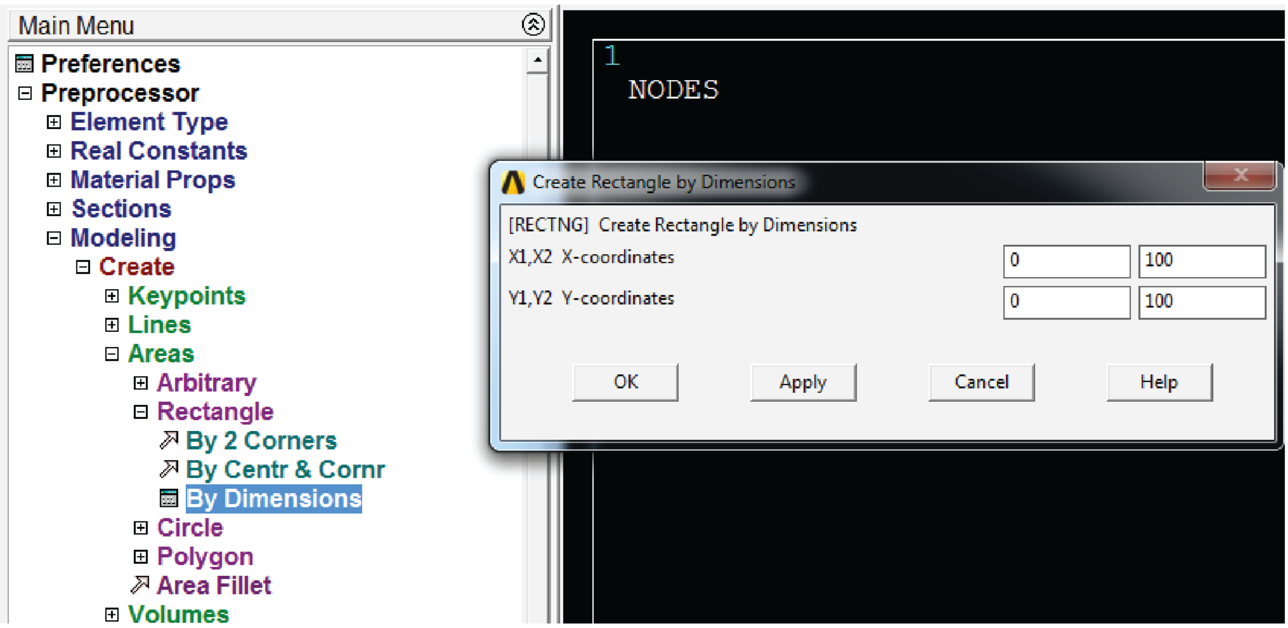

Figure 3.14: Creation of the plate's geometry.

B. Definition of the orthotropic material properties.

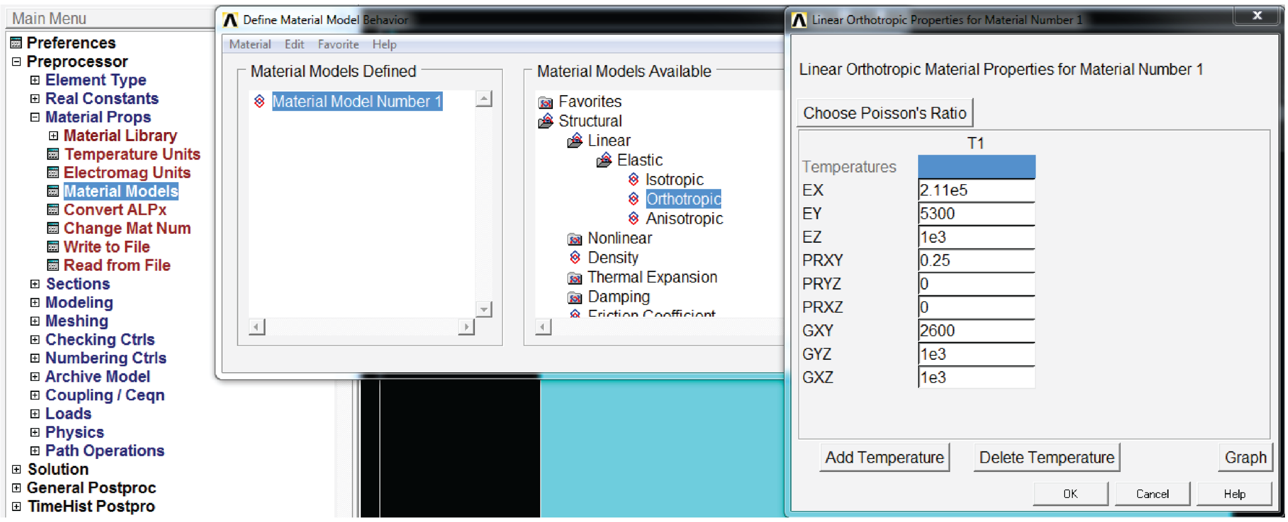

Figure 3.15: Defining properties of the orthotropic material.

C. Selection of the finite element type - multilayer shell:

Structural $\longrightarrow$ *Shell* $\longrightarrow$ *8node 281* **(fig. 3.16)**.

To present results for all layers, we need to select adequate type of saving in the element's options:

Storage of layer data K8 $\longrightarrow$ *All layers.* **(fig. 3.17)**.

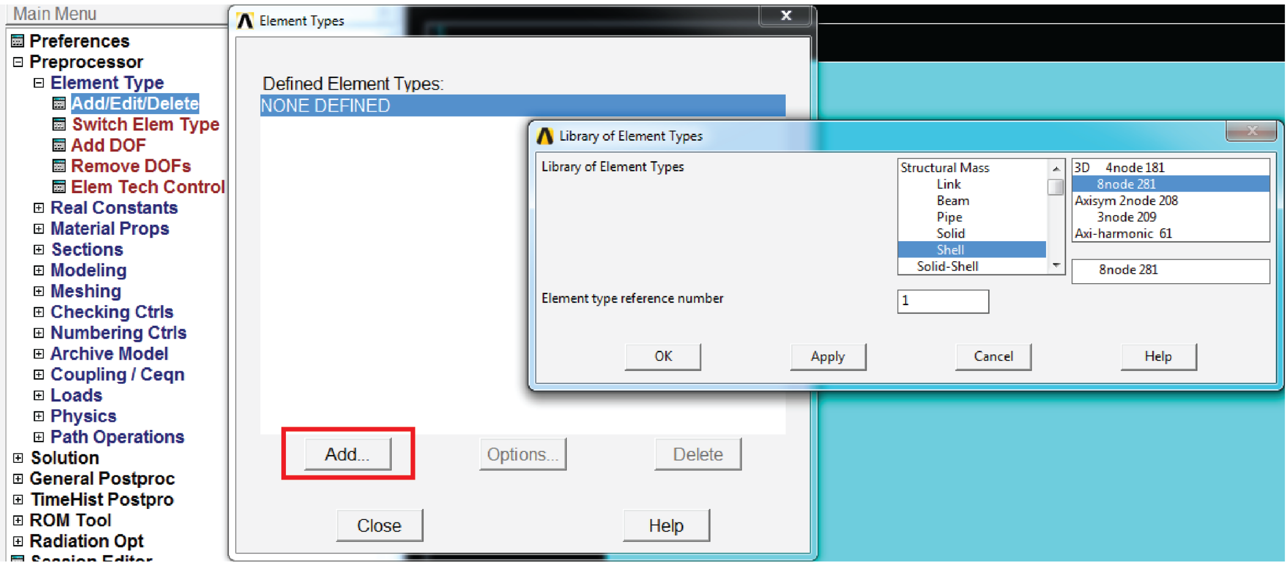

Figure 3.16: Selection of the Shell element.

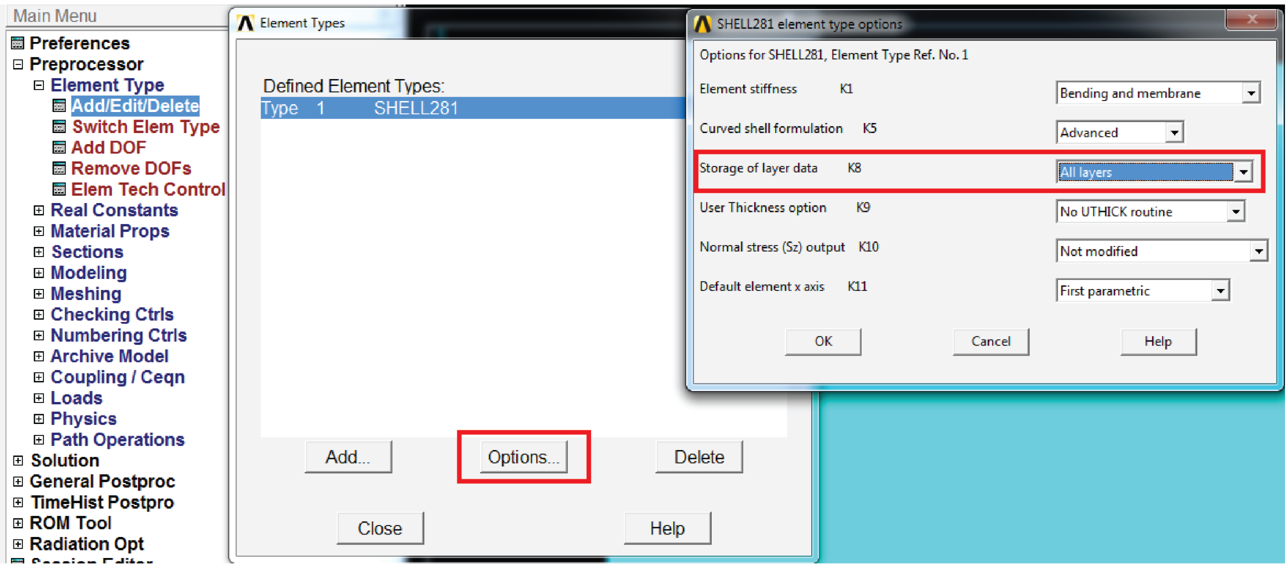

Figure 3.17: Selection of the element's results saving option.

D. Definition of thickness, material model and the directions of orhtotropy for all of the layers **(fig. 3.18)**.

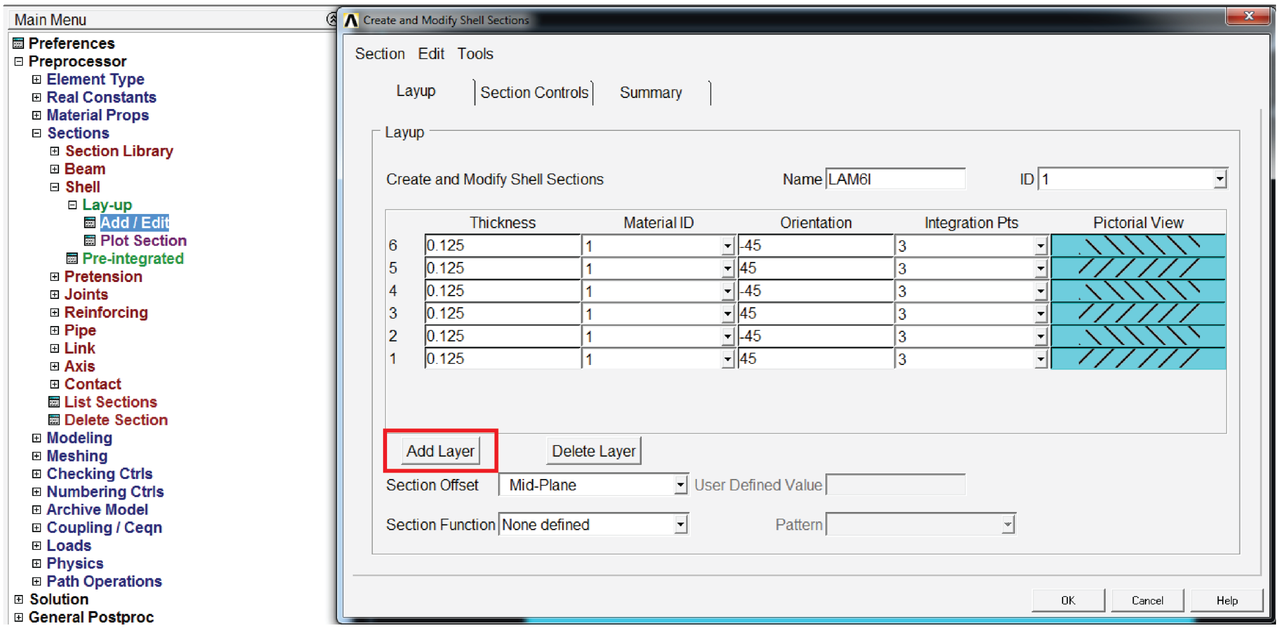

Figure 3.18: Creation of the laminate lay-up. To add next layer use the button *Add Layer*.

E. Finite element meshing

After the mesh is created the lay-up of the laminate can be illustrated in two ways. Schematically:

Sections $\longrightarrow$ *Shell* $\longrightarrow$ *Lay-up* $\longrightarrow$ *Plot Section*. **(fig. 3.19A)**,

or graphically, with each layer's thickness shown:

Plot Ctrl $\longrightarrow$ *Style* $\longrightarrow$ *Shape and Size* $\longrightarrow$ *Eshape on*. **(fig. 3.19B)**.

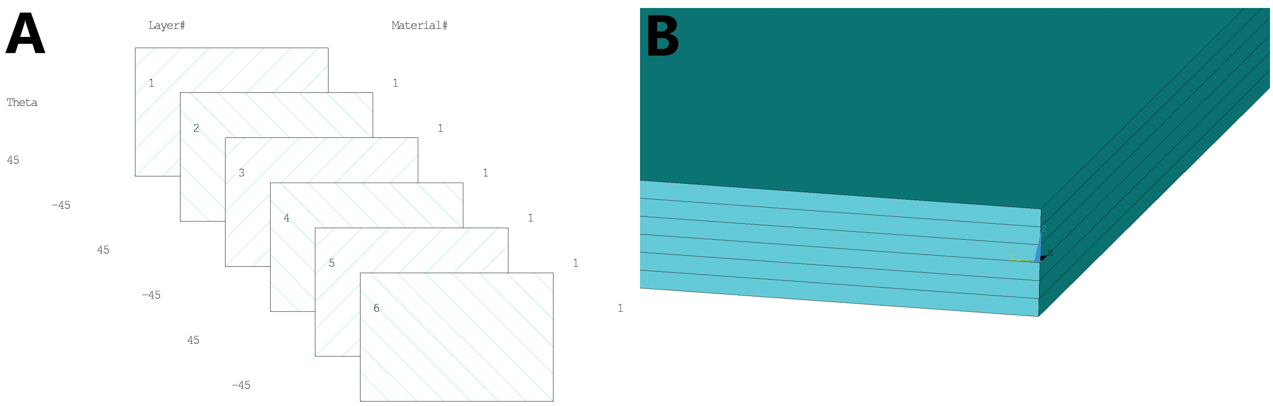

Figure 3.19: Two methods of laminate presentation. **A** - schematically with the directions of orthotropy. **B** - graphically with visible thickness of each layer.

F. Definition of the boundary conditions. Constraints must provide free deformation of the model (i.e. they must be statically determinate). For example $u_x = u_y = u_z = 0$ at the point **A**, $u_x = u_z = 0$ at **D** and $u_z = 0$ at **B** **(fig. 3.20)**. As a load, we apply tensile pressure on the edges **AD** and **BC** equal to $10\frac{N}{mm}$.

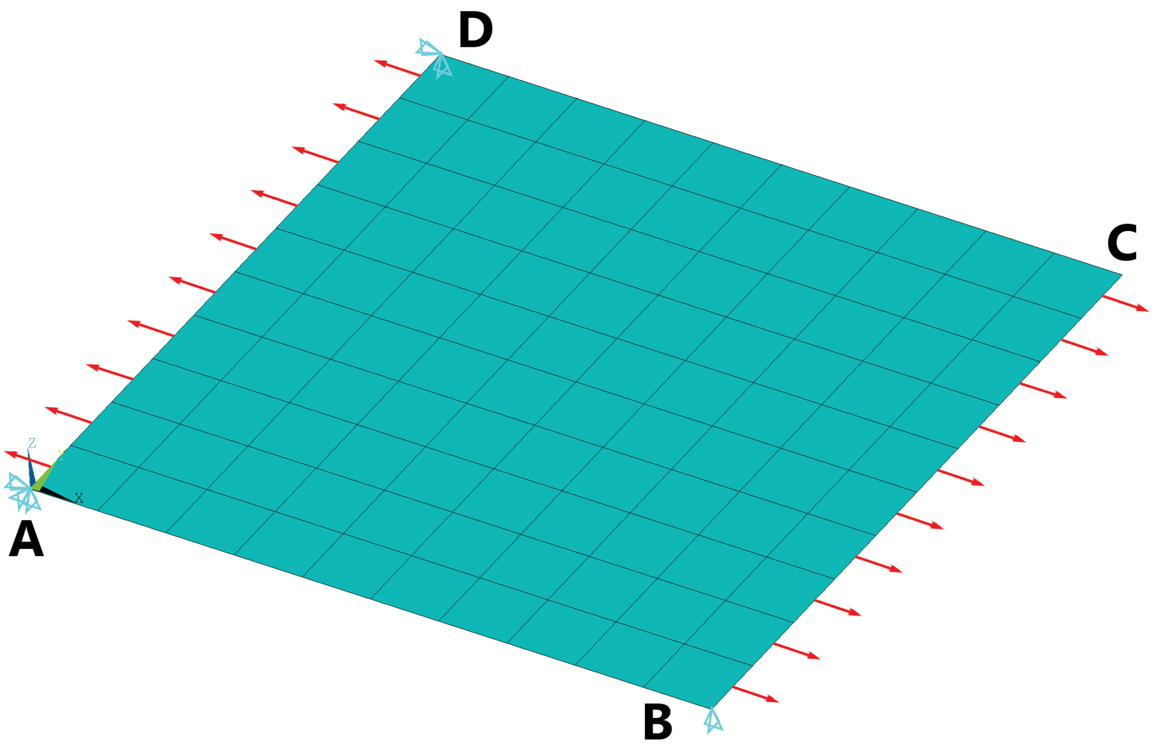

Figure 3.20: Completed FEM model.

Solution: Running the solution: *Solve* ⟶ *Current Load Step*.

General Postprocessor:

Evaluation and comparision to the analytical solution of the displacements obtained from the analysis.

To show stress and strain in the selected layer, we use *Options for Output* command with option *Specified Layer number* filled accordinlgy **(fig. 3.21).**

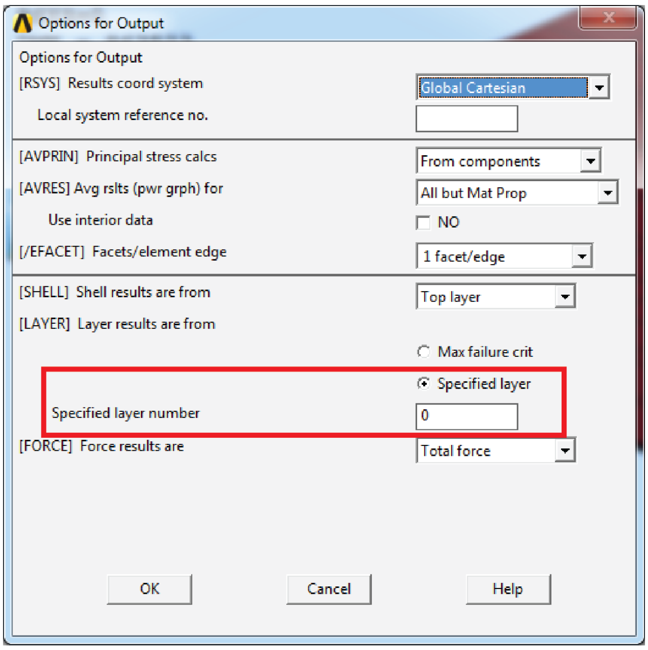

Figure 3.21: Options to show results in individual layers.

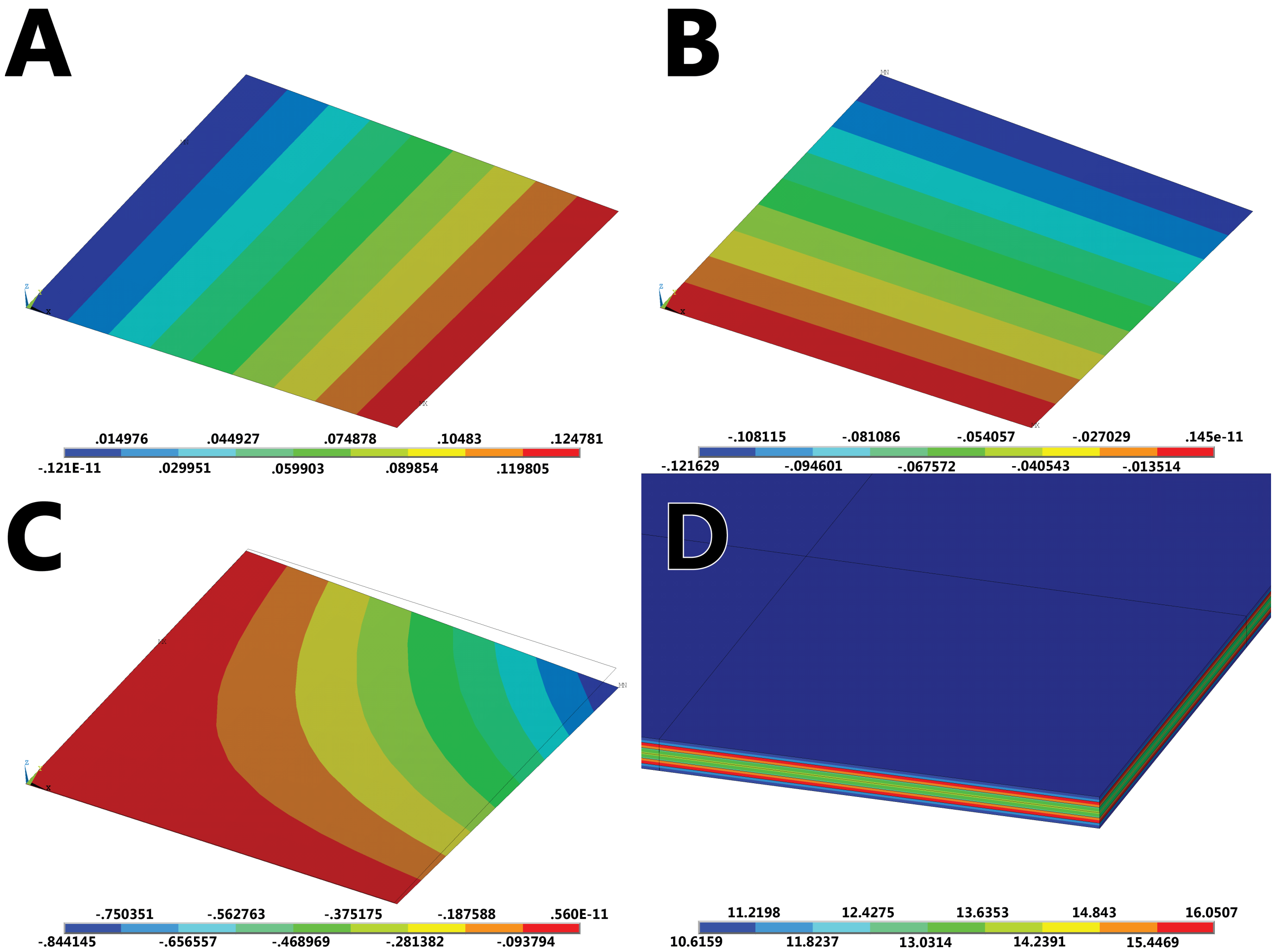

Figure 3.22: Results obtained from calculations. **A** - displacements in x direction [mm], **B** - displacements in y direction [mm], **C** - displacements in z direction [mm], **D** - stresses in x direction - σ_x [MPa] in the whole laminate.

4 Further tasks

1. Perform the analysis from section 3.1 with a different angle of the orthotropy direction. Compare obtained results with the analytical values from the table 2.1.

2. Perform the analysis of the tensile test of a square plate ($100mm$ x $100mm$) with central circural hole ($R = 25mm$) for two directions of orthotropy: $\alpha = 0°$ and $\alpha = 45°$. Calculate the strenght indexes for four avaiable yield hypothesis (fig. 3.13).

3. Perform the analysis for a plate laminate with a different lay-up. Calculate the displacements for the x, y and z directions.